History and critical analysis of fifteenth and sixteenth century nautical tables.

Siebren van der Werf[1]

The Netherlands

Abstract

Analysis of fifteenth and sixteenth century solar tables which have been used in navigation and of declination tables derived from them, reveals that much of the existing and accepted history writing needs revision. In particular: the celebrated astronomical tables of Abraham Zacut contain systematic and accidental errors by which later tables may be identified as derived from them. The tables of Pedro Nunez and Martín Cortes are not their own, but have been copied from those of Philipp Imsser, which are usually, but incorrectly, attributed to his predecessor Johannes Stöffler. William Bourne used a wrong conversion table to calculate his declinations and had to manipulate his data to hide it. Pedro de Medina's Dutch translation by Marten Everaert has declination tables that fit in neither the Alfonsine nor the Copernican scheme and are inconsistent within themselves. Also the declination tables of Lucas Jansz. Waghenaer need amendment and those of Willem Jansz Blaeu are older than he suggests himself.
The absolute accuracy of declination tables is assessed in retrospect by comparison with modern celestial mechanics programs.

Keywords

Declination tables, astronomical ephemerides, Alfonsine Tables, Prutenic Tables, history of navigation, accuracy assessment, fifteenth and sixteenth century

Introduction

The need for declination tables of the sun by which it would be possible to find one's latitude in open sea became urgent near the end of the fifteenth century. It was the age of the maritime expansion of seafaring nations in search of alternative trade routes to the East and the discovery of new lands. Portugal and Spain led the way.

Finding your latitude by taking the height of the Pole star was a known technique, but on the southern hemisphere it is below the horizon. To navigate by the height of the sun, mariners needed tabulations of the solar declination. Astronomers would, however, not tabulate declinations, but instead the sun's places along the ecliptic. Converting these ecliptic longitudes (λ) to declinations (δ) is relatively easy, at least for the sun:

$$\delta = \arcsin[\sin(\varepsilon)\sin(\lambda)] \qquad (1)$$

where ε denotes the obliquity angle between the ecliptic and the equator.
Since this article is meant to address also a nautical readership, I will henceforth denote δ as *DEC* and λ as *ECL*, in line with nautical custom of using three-letter abbreviations.

[1] E-mail: vdwerf@kvi.nl or s.y.van.der.werf@rug.nl

The procedures of how to calculate ecliptic longitudes were given in the *Alfonsine Tables*, which date back to 1252 and were a continuation of the astronomy of Ptolemy.[1] Halfway the sixteenth century, in 1551, another set of tables was published by Erasmus Reinhold: the *Prutenic Tables*,[2] based on the work of Copernicus.[3]

Declination tables were never *ab initio* calculations but were instead based on existing astronomical ephemerides. Exactly how they were made and from which ephemerides, is known or easy to verify in some cases. Identifying the connection is, however, not obvious in all cases, because writers copied freely from each other in those days and quoting your sources was an exception rather than the rule.

In this article I attempt to make these identifications and to assess the accuracy and consistency of those ephemerides and declination tables that were most popular in the history of sixteenth century navigation. In the first place that is an investigation of how accurately and truthfully tables have been made after either the Alfonsine or the Prutenic prescriptions. In the second place I will assess the absolute accuracy of the declination tables by comparing them with calculations using modern astronomical software.

The Alfonsine Tables

Originally handwritten, the Alfonsine Tables first appeared in print by the end of the fifteenth century, in 1483 compiled by Johannes Danck de Saxonia[4] and printed in Venice. An update by Johannes Lucilius Santritter[5] followed in 1492, also printed in Venice.

The calculation of the sun's true place along the ecliptic required finding first the mean places of the sun and the apogee from their *radix*, usually taken as January 1, following the birth of Christ and at mid-day in the time of Toledo. The apogee should then be corrected for a periodically accelerated and decelerated epicycle motion of the eight's sphere, that of the fixed stars. Then, via the table of the *equatio solis*, the difference between the sun's mean and true longitudes, the true place of the sun is found.

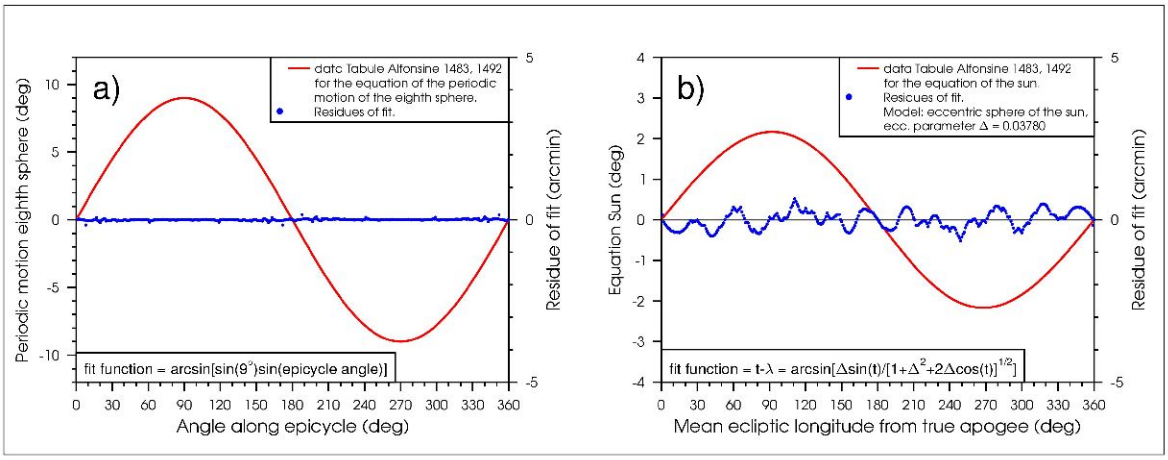

Figure 1. a) The "motus accessus & recessus" epicycle correction (period 7000 years) to be added to the mean progression of the apogee (period 49000 years). b) The difference between the mean and the true ecliptic longitude of the sun (*equatio solis*) over the year, comparing its tabulation in the Alfonsine Tables with a rigorous calculation.

Figure 1 shows these corrections and their corresponding fits. The epicycle correction for the eighth sphere is a pure product of sines and in the fifteenth century this could be accurately computed, as shown in Figure 1(a). Figure 1(b) shows the fit of the tabulated *equatio solis* with its analytical formula. Here Δ is the eccentricity, the displacement of the center of the Sun's orbit from the Earth, relative to the radius of the solar orbit. The value that best reproduces the equatio solis of the Alfonsine Tables is Δ = 0.03780. The fit shows some irregularities, illustrating that a mixed expression of trigonometric functions and powers could not yet be evaluated with the same accuracy as a purely trigonometric function. In comparing tabulated solar longitudes with calculated ones, I will further use interpolation in the tables, which is of course what the practitioners did themselves.

If declinations are required, then the fourth step is to apply the conversion as given in eq. 1 for the obliquity of one's choice, because the Alfonsine Tables themselves do not provide it.

Johannes Regiomontanus

Johann or Hans Müller (1436 - 1476), born in Königsberg, Lower Franconia in the north of Bavaria, became known under his adopted writers name Johannes de Monte Regio. The name Regiomontanus, by which he is remembered, is of later date. A good and concise text on his life and works is Edward Rosen's contribution in the Complete Dictionary of Scientific Biography.[6] Most important for us are his contributions that have a connection with navigation. He produced daily ephemerides for the sun,[7] the moon and the planets for the years 1474 – 1506 in a format that would become the standard for later centuries. The meridian was chosen as Nuremberg, 84 minutes in solar mean time East of Toledo by the "Table of Cities" of the Alfonsine Tables. Today we know that the difference in longitude between the two cities is 16º 49′, which translates in a time difference of only 1h 7m. This illustrates that in the fifteenth century the circumference of the Earth was underestimated by some 30%. It may have been one of the reasons that Columbus thought he had already reached the Indies.

For seafarers there was also Regiomontanus' *Tabulae directionum profectionumque*,[8] 'tables of directions', with elaborate tabulations for the conversion of ecliptic longitudes to declinations. In the same year, 1474, he printed his *Kalendarium*,[9] which contains among many other things, a "quick and dirty" method for finding solar longitudes by extrapolation from a root year. Because of its importance in nautical history I will deal with it in some detail.

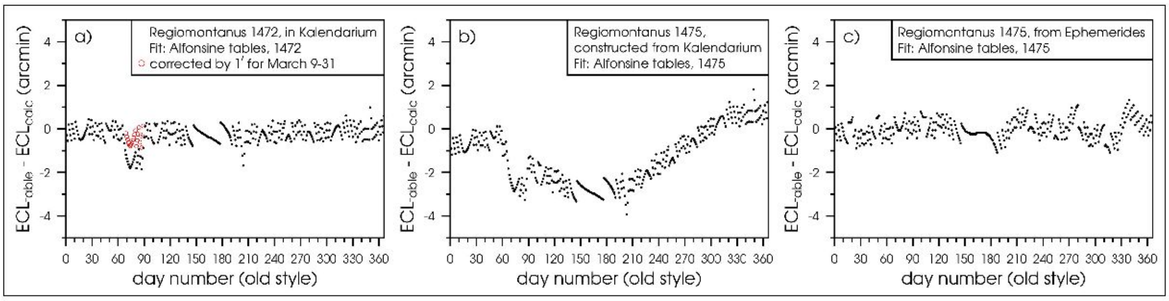

Figure 2. Differences of tabulated solar ecliptic longitudes (*ECL*) and Alfonsine Table based calculations. a) For the root year 1472. For March 9-31 data appear systematically 1´ low. Corrected data are in red. b) *ECL*'s for 1475, obtained via the *Kalendarium* extrapolation method. c) *ECL*'s for 1475 from Regiomontanus *Ephemerides*.

The root year is 1472. Figure 2(a) shows a fit with a calculation via the Alfonsine Tables. Since the tabulated *ECL*'s are rounded off to the nearest arcminute and the calculation is not rounded, the difference plot shows a horizontal band, 1′ wide and centered around zero, as it should.

To obtain *ECL*'s for a later year you then add an overall shift, to be found from a separate table. For common years (no leap year) the procedure is then to leave out 29 February and to subtract 1˚ from the solar longitudes from March till the end of the year.

Figure 2(b) shows the data that result from this prescription for 1475. The clearly visible discrepancy is caused by the fact that shifting the *ECL*'s of a root year affects the equinox and the apogee differently. For 1475 the prescribed overall shift is 16′. This additional 16′ backshifts the equinox by about a quarter of a day, as counted from the beginning of the year. But leaving out the leap day places it back to about 18 hours later on the calendar. Indeed, by the Alfonsine Tables the equinox was at 10 March 11:37:27 PM in 1472 and in 1475 at 11 March 05:05:09 PM, in the solar time of Nuremberg.

For the apparent place of the apogee the extrapolation has a different effect. Adding or subtracting an overall constant does *not alter the day-to-day differences* in solar longitudes. Having left out the leap day, the data suggest that the apogee, the point where the sun's pace is slowest, is now one full day earlier as counted from the beginning of the year, hence still on the same calendar day, 12 June, as in 1472, and at the same time, 22:14:05 PM. The true Alfonsine Table calculation puts it at 13 June 1475, 16:23:27 PM, about eighteen hours later.

Figure 2(c) shows the table for 1475 as Regiomontanus gives in his *Ephemerides*. It illustrates that he did not produce these ephemerides by his Kalendarium procedure, but by proper calculations for each year separately.

Summarizing on the Kalendarium method: it is correct for finding the equinox in later years, but it leaves the time of the apparent apogee as it was in the root year.

The obvious advantage of the method is that you need only one solar table. Moreover, when the shifted *ECL*'s are used to produce declinations, the apparent misplacement of the apogee is hardly a problem, because the maximum deviations in *ECL* are around the apogee and the perigee which are close to the summer- and winter solstices, where declinations vary slowest. Almost a century after Regiomontanus, Martin Cortes used the method again in his *Breve Compendio*.[10]

Abraham Zacut's *Almanach Perpetuum*.

Abraham Zacut's *Almanach Perpetuum Coelestium Motuum*, the "eternal almanac of the motions of the heavens" has been of great influence on the developments in Portuguese and Spanish navigation. Originally written in Hebrew, the *Almanach*'s Latin translation by José Vizinho was published in 1496.[11] A reprint with corrections appeared in Venice, in 1502.[12] Tables for the sun are given for four years, 1473 – 1476, but with a recipe for how to re-use them for later years. To make the extrapolation to later years more reliable, Zacut chose to tabulate his ecliptic longitudes to an accuracy of one second of arc. His calculations were made for the meridian of Salamanca, eleven minutes West from Toledo in solar mean time. Furthermore, he lets the year begin on 1 March.

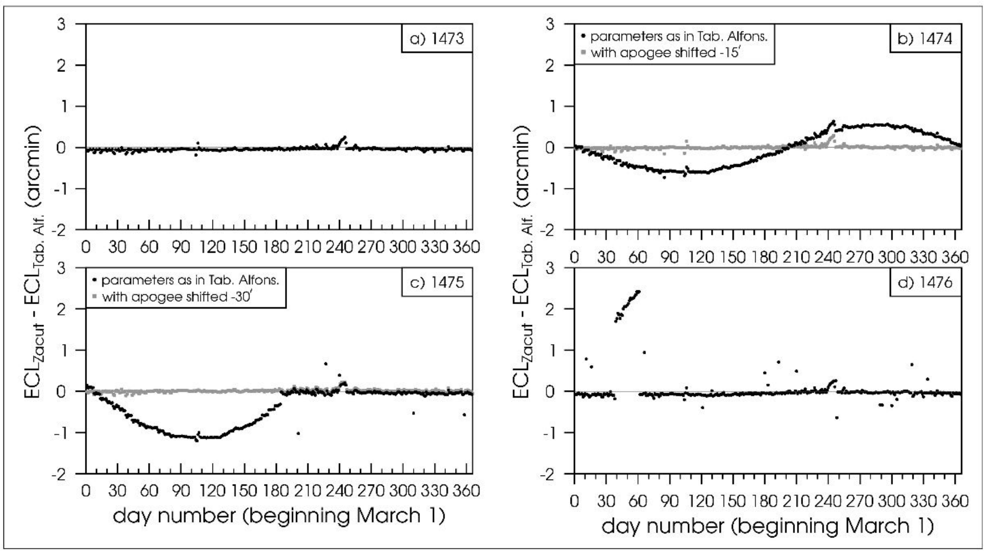

Figure 3. Residues of Zacut's solar places (1473 – 1476) upon fits with direct calculations via the Alfonsine Tables for the meridian of Salamanca. For 1474 and 1475 also with shifted apogee.

Figure 3 compares Zacut's tabulated *ECL*'s with the prescriptions of the Alfonsine Tables in the form of differences between the two. For 1473, Figure 3(a) shows excellent agreement. The same agreement was found by Chabás and Goldstein,[13] who recomputed the *ECL*'s only for March 1473.

A remarkable systematic deviation shows up in the second year, Figure 3(b), where the ecliptic longitude of the apogee seems to have been chosen about 15′ too small, and in the first half of the third year, Figure 3(c), where it seems too small by about 30′. Inspection of these tables reveals that they are not independent calculations, but just shifted versions of 1473, subtracting 14′ 21″ for 1474 and 28′ 41″ for the first half of 1475. As shown above for Regiomontanus' Kalendarium method, this procedure correctly adjusts only the equinox, not the apogee. When comparing these shifted data with true calculations this shows up as a cosine-form deviation around the place of the apogee. Evidently, Zacut discovered his mistake and from halfway the third year onward the equation of the sun has been correctly calculated.

In the fourth year, Figure 3(d), an unfortunate glitch occurs from 8 April to the end of the same month. This mishap has most likely occurred at the printer's office. While the column that gives the degrees continues normally, the arcminutes and arcseconds for April 8 have been repeated from April 7 and this shift of one day is restored only at the beginning of the next month. The resulting error of about 2′ falls in a time of the year where the declination changes rapidly and it is by this 'April glitch' that later tables may be identified as based on an extrapolation of Zacut's ephemerides.

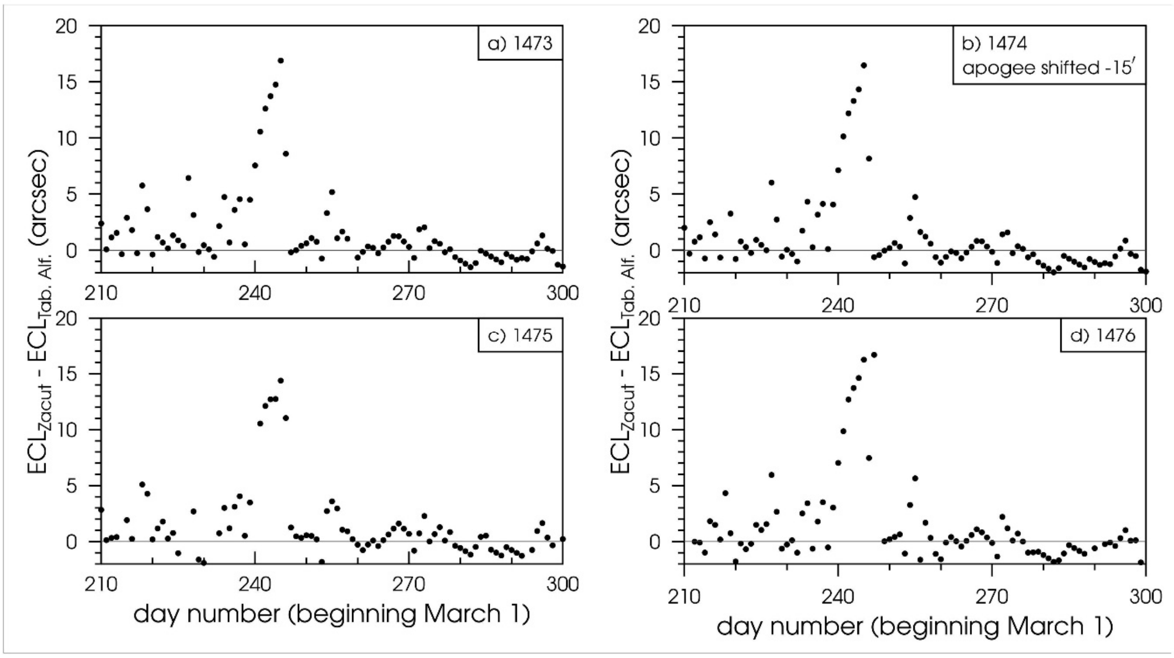

Figure 4. Residues of Zacut's solar places (1473 – 1476), enlarged to bring out the October irregularity.

A tiny irregularity occurs in all four years in the last week of October. The discrepancy becomes maximally 15-17 arcsecs. This is shown in Figure 4. The effect occurs between 42-52 days before the sun passes through the perigee. If it would stem from the table Zacut used for the *equatio solis*, then it should also be there 42-52 days after passage, which would be the last week of January and the first days of February. There is nothing special there, and the October irregularity remains mysterious.

Extrapolation to later years is simple. The length of the tropical year is not precisely 365 days and 6 hours, but just a bit shorter. By the mean solar motion as the Alfonsine Tables prescribe it, the sun covers in four years (1461 days) 1′ 45″ 48‴ in excess of four full circles along the ecliptic. This little excess, rounded off to 1′ 46″, is then the difference that you have to add to a tabulated ecliptic longitude to get its value on the same calendar day four years later. Zacut gives this accumulated difference for up to 34 such four year periods and he calls it *tabula equatio solis*, the table of the equation of the sun. The name was a bit unfortunate, because the same phrase was also used for the difference between the sun's mean and apparent longitude. Yet, the name stuck, especially in nautical handbooks, and the shift to be applied to the ecliptic longitude for use in later years became known as the *equation of the sunne*. With his so extrapolated *ECL* a late fifteenth century navigator would then enter another table, the *tabula declinationis*, and by interpolation find the declination of the sun.

The Regiment of Munich

The oldest known printed declination tables are in a small booklet, discovered around 1890 in the Royal Library of Munich. It was printed in Lisbon, but the name of the author and the year are not known. Joaquim Bensaúde[14,15] made an extensive study of the Munich regiment and included a facsimile of the original. It is also included in De Albuquerque's *Os guias nauticos de Munique e Évora*.[16]

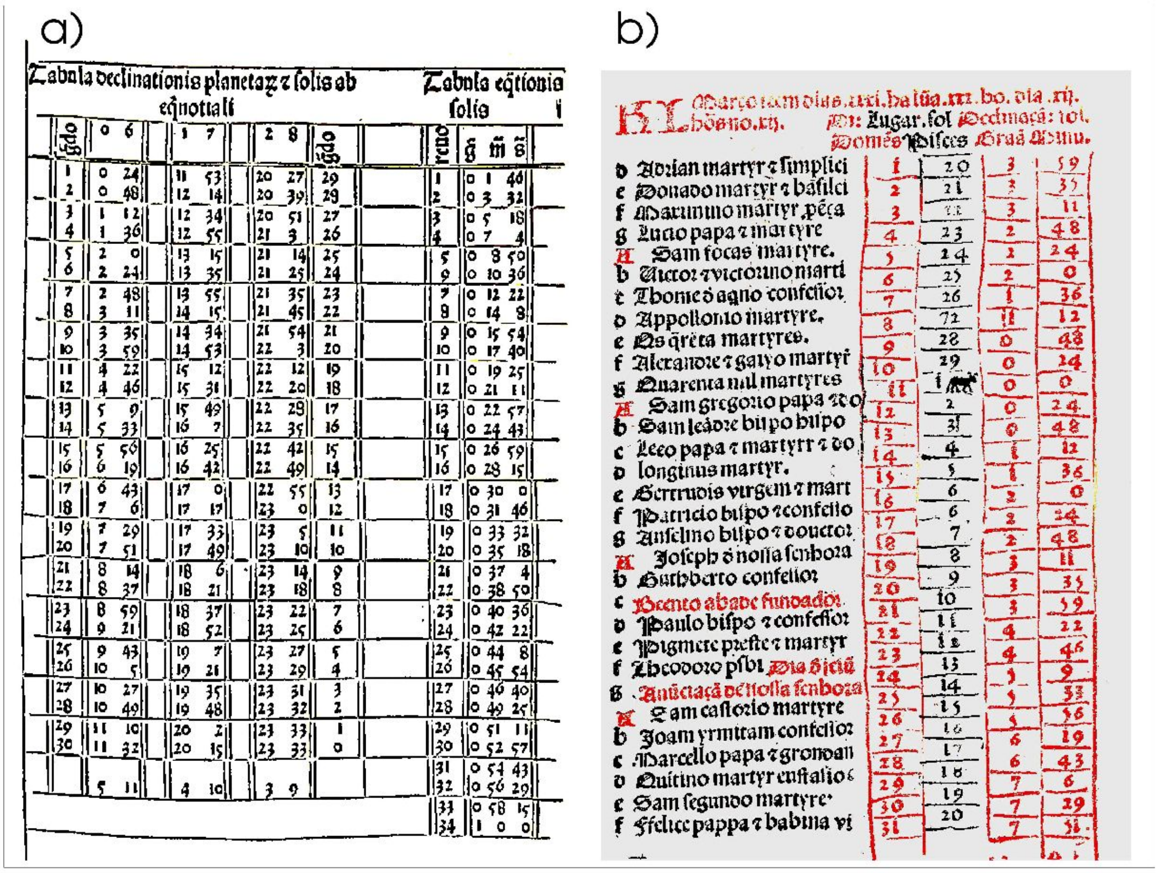

Figure 5. a) Tables from Zacut's *Almanach Perpetuum*. Left the *tabula declinationis*, right the equation of the sun. b) Solar places and declinations for March in the Munich regiment (from Bensaúde, 1914).

Instructions for measuring the sun's height with an astrolabe or a quadrant and for deducing one's latitude at sea are given in the *Regimento do estrolabio e do quadrante*, which contains the declination tables. The term "Regiment" for a set of nautical tables and the instructions for how to use them, became also common in sixteenth century English nautical language. Also included in the Munich booklet is the Regiment of the North Star and an introduction into astronomy and cosmography in a Portuguese translation of *De Sphaere Mundi*,[17] on the spheres of the world. The Latin original was written by a thirteenth century English monk, Joannis de Sacrobosco (John Holywood) and it was widely used in schools of navigation and universities.

The declination tables are given for only one year, which is a leap year, and like in Zacut's *Almanach*, they begin with March. The tables are highly schematic, but brilliant in their simplicity.

Figure 5(a) shows Abraham Zacut's conversion table for ecliptic longitude to declination, as it is found in his *Almanach*. It is based on a maximum declination of 23º 33′. Still in the left panel, but on the right-hand side of it, is the equation of the sun, the accumulated shift in ecliptic longitude over four years periods, tabulated for one such period to 34 of them.

Figure 5(b), shows the declination table from the Munich regiment for March. For each day it mentions the Saint whose name day it is. Next, in the second column of numbers are the ecliptic longitudes, starting out at 20º in Pisces and entering into Aries on the eleventh. The ecliptic longitudes are all rounded off to their nearest degree. Hence the declination, given in degrees and minutes in the last two columns may be directly taken from Zacut's table and comparison shows that indeed they are identical. The sign of the declination is not given, but it is understood that declinations are South before the equinox and North after.

Even if the tabulated ecliptic longitudes have been rounded off, one may still try to find a best agreement with an extrapolation from Zacut's ephemerides for 1476. By a least squares fit to all 366 data points, I find a shift of eight or nine four years periods, both with about equal likelihood. This would date the tables as made for either 1508 or 1512 in agreement with Portuguese authors on nautical history, Bensaúde,[15], De Albuquerque[16] and Pereira da Silva,[18] who have estimated that the Munich regiment would date from around 1509.

By rounding off the ecliptic longitudes to the nearest full degree an uncertainty of half a day at maximum is introduced. Declinations vary most rapidly near the equinoxes where they reach a change of 24′ per day. Hence, the error in the tabulated declinations is about 12′ at worst. Given the accuracy with which a solar height could be taken, this was still quite acceptable. Recent researches on the accuracy of the astrolabe, by Köberer,[19] Knox-Johnston[20] and De Hilster[21] and references quoted therein, indicate that even with a perfect instrument measurements taken on land by experienced observers would not have been significantly more accurate, as judged by their reproducibility (standard deviation). At sea the statistical spread may easily have been twice as large.

The Regiments of Évora , João de Lisboa and André Pires

Declination tables for four years appeared in print about ten years later. They are found in the Regiment of Évora, so named after the city where the only known surviving copy was found. It does not mention the author nor the year of print. Besides the tables for declination, it gives ecliptic longitudes, but only for the leap year. These tables have been reproduced in De Albuquerque's *Os guias náuticos*.[16] The *Livro de Marinharia* of João de Lisboa[22] has the same declination tables, but gives no ecliptic longitudes. The declinations for the leap year are also found in the *Reportório dos tempos* of Valentim Fernandes.[23] The year of print of this last work is known. Fernandes himself was the printer and he finished it in 1518, shortly before his death. More elaborate is the *Livro de Marinharia* of André Pires.[24] It has two groups of four-year tables, both for declinations and for ecliptic longitudes. The declinations are the same in both groups and they are the same as those of the Évora regiment and in the guide of João de Lisboa. The tables are given in De Albuquerque's book *O Livro de marinharia de André Pires*.[27]

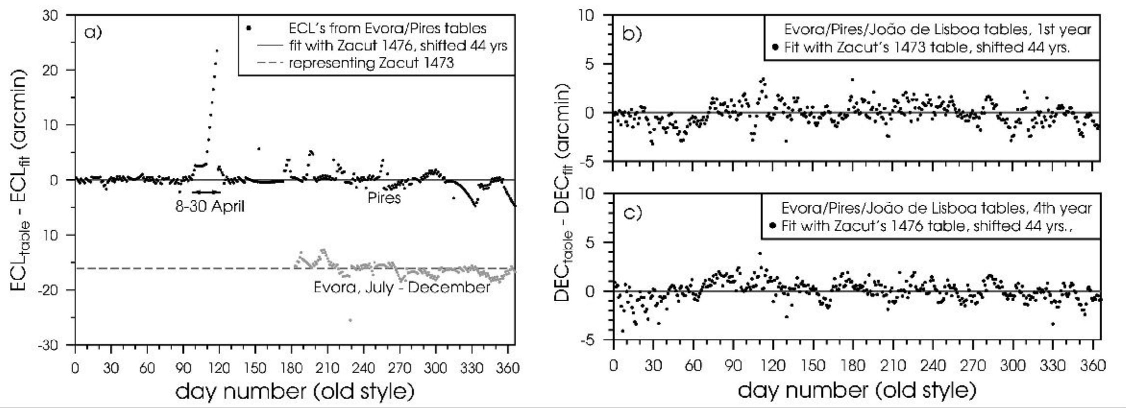

Figure 6. a) Ecliptic longitudes for the leap year in the Évora and Pires (group 2) tables, compared with Zacut's solar longitudes. b) Declinations for the first year and c) for the leap year, compared with values derived from Zacut.

Ecliptic longitudes for the leap year are shown in Figure 6(a). The first half of the *ECL*-table of the Évora regiment, January – June, is identical to that of the second group of Pires. It appears to have been obtained by shifting the Zacut 1476 tables over 44 years. It even reproduces the 'April glitch', the 2′ deviation from 8-31 April that is encountered in Zacut's tables. In the Évora regiment, the *ECL*'s for July – December correspond to 1473, also shifted over 44 years. In the guide of Pires they have correctly been based on 1476.

The declination tables apply to 1517 – 1520. Fits with calculations via Zacut's tables are shown in Figures 6(b) and 6(c). The spread of the residues, as measured by the standard deviation, or root mean square error, is just above 1 arcminute.

One small detail must be solved: in the Évora regiment the leap year comes first and the others are indicated as the first, second and third year *after* the leap year. Analysis under this scenario gives significantly worse fits, measured again by the standard deviations, making this hypothesis much less likely. The leap year comes last, not first, and the quadruplet is 1517 - 1520.

Declination tables for 1529 – 1532 of De Enciso, Faleiro and De Medina

While the Évora/ João de Lisboa/ Pires declination tables were already quite accurate, the next generation tables was arguably the finest of the sixteenth century. We find them in the 1530 edition of De Enciso's *Suma de Geographia* printed in Seville by Jacob Cromberger.[26] They are also included in Francisco Faleiro's *Tratado del esphera y del arte del marear* of 1535.[27] Not accidental for sure, because the printer was again Cromberger. The book that made these tables popular all over Europe was Pedro de Medina's *Arte de Nauegar* from 1545[28] and especially its French edition of 1554, translated and extended by Nicolas de Nicolai.[29] An English translation by John Frampton[30] came out in 1581 and a year earlier a translation in Dutch, by Marten Everaert and Michiel Coignet had been printed in Antwerp.[31] In the latter version the original declination tables are replaced by different ones that I will discuss later.

The tables have been made for the years 1529 – 1532 by extrapolation from Zacut's ephemerides, a shift of no less than 56 years. To illustrate how accurately they have been made: the standard deviation over the full four year period is 0′.43, to be compared with the theoretical minimum for rounding off to the nearest arcminute, which is 0′.29.

Just because of their extreme precision, two characteristic features show up which prove that indeed these declination tables were made by extrapolation from Zacut's ephemerides. Most obvious is the April glitch, where in Zacut's tables the minute and arc seconds have been shifted by one day for 8 – 30 April in his fourth year. Its effect is not seen in the fit, but when the ecliptic longitudes are set to what they were supposed to be, it *does* show up, as illustrated in Figure 7(b).

The other characteristic is in the first two months of the first year. Zacut lets his years begin with March and end with February. The declination tables begin with January and the compilers had therefore to take January and February of their first year from the last months of Zacut's fourth year. In doing so, they evidently forgot to shift Zacut's *ECL*'s back by 1′ 46″, the equation of the sun over one extra four year period. Correcting for this omission shows up as a discrepancy. See Figure 7(a).

All this illustrates that these declination tables were indeed made from extrapolated Zacut ephemerides and that these were accepted at face value.

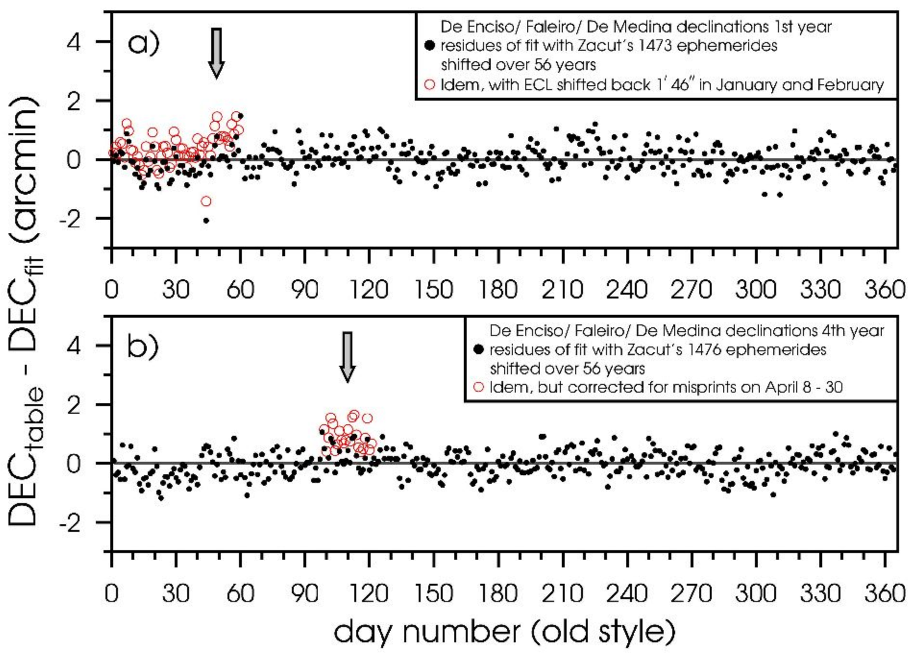

Figure 7. Declinations from De Enciso (1530), Faleiro (1535) and De Medina (1545) for the first year (a) and the leap year (b). Places where their origin as derived from shifted Zacut ephemerides shows up are indicated by arrows and by a "corrected" calculation.

Pedro Nunez and the solar tables of Philipp Imsser

In 1537 Pedro Nunez published his famous book *Tratado da sphere*.[32] It contains an extended translation of Sacrobosco's *De Sphaerae Mundi* and a treatise on the motions of the sun and the moon. Further a translation of Ptolemy's first book on geography and a treatise on the construction of sea charts. For the mariner who wants to find his latitude by taking the height of the sun, there are tables of ecliptic longitudes for the years 1537 – 1540 and another table for converting them to declinations. For use in later years, the equation of the sun is the usual shift in *ECL* of 1′ 46″ for every four years. It is the same procedure as prescribed by Zacut, but for a later set of root years.

An important difference is that Nunez' declination table is based on a maximum of 23º 30′, the same as adhered to by the school of Regiomontanus, while until then all Portuguese and Spanish tables had used Zacut's value of 23º 33′. The ecliptic longitudes that Nunez gives are systematically about 3′ lower than found via extrapolation of Zacut's tables and Luciano Pereira da Silva[18] has hypothesized that the publication of Nunez' *Tratado* had met with some delay and that in reality these tables had been made for an earlier set of years.

The solution is, however, much simpler. The years are right, but the meridian is not Salamanca or Lisbon as one would have expected even though Nunez does not mention it. The tables are for the meridian of Tübingen and the author is not Nunez, but Philipp Imsser.[33] Immser had written ephemerides for the years 1532-1551, following the format and tradition of Regiomontanus. His predecessors in this series were Johannes Stöffler and Jakob Pflaum,[34] who had made tables for 1499-1531, for the meridian of Ulm.

Johannes Stöffler had died and Philipp Imsser succeeded him as professor in mathematics and astronomy in Tübingen. Figure 8(a) shows the title page with Stöffler's picture on it. This is probably the reason that up till the present day these tables are generally not recognized as Imsser's but are attributed to Stöffler. The title itself, however, states clearly that these tables are not *by* Stöffler, but *in continuation* of his work. And the address to the Emperor is signed: Philippus Imsser, Argentinensis. This last detail tells us that he was from Strasbourg.
John Roche[35] has drawn attention to these ephemerides. Not mentioning that they are fully identical, Roche writes that Nunez' ephemerides agree with those of Stöffler (read: Imsser).

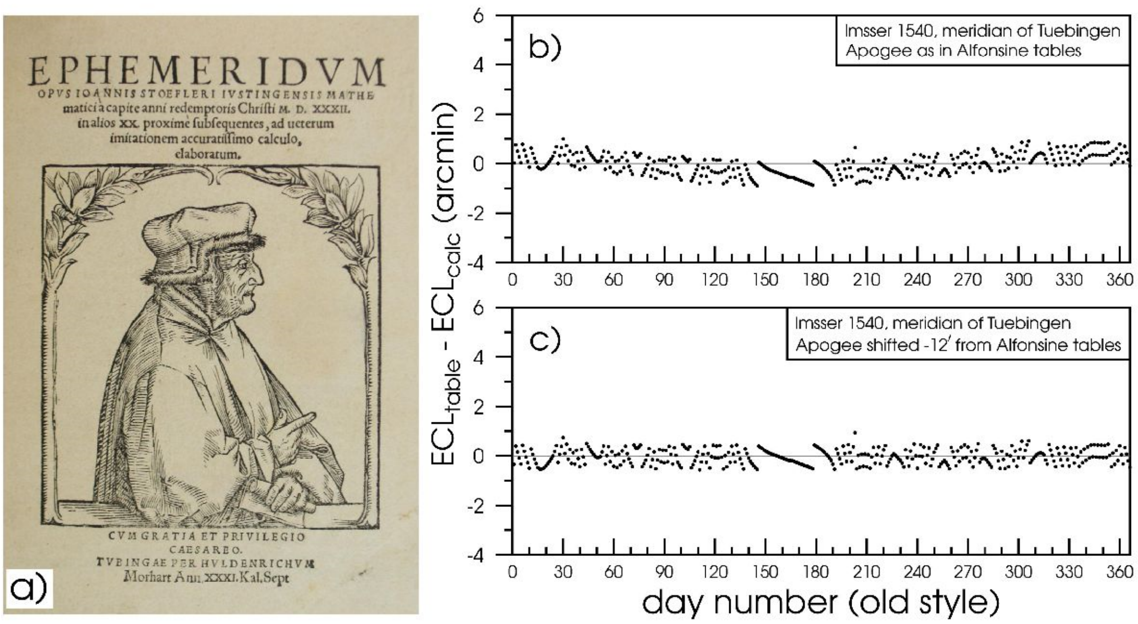

Figure 8. a) Title page of Imsser's ephemerides for 1532-1551. b) Residues for *ECL*'s of 1540 upon fits with the Alfonsine Tables. c) Idem, but with the apogee shifted.

Imsser's ephemerides differ in a few aspects from those of Stöffler and Pflaum. In the first place he transferred the standard meridian from Ulm to Tübingen, which in his "Table of Cities" he puts at 79 minutes East from Toledo. Yet his tables tell a different story. By analyzing his full solar tables for 1532, 1537, 1540, 1545 and 1548, I find that invariably they require a distance of only 60 minutes. Moreover, he systematically uses a place of the apogee that is about 10′ - 12′ lower than what the Alfonsine Tables prescribe. The periodic motion of the eight's sphere that enters in the calculation had throughout been a subject of debate and it may well be that Imsser had a different view than his predecessors. Analyses for 1540 (also the last year in Nunez' *Tratado*), with and without this apogee shift are shown in Figures 8(b) and 8(c).

These are also the conclusions of Pietro Pitati (Petrus Pitatus) in his *Almanach Novum*.[36] Pitati gives the years 1544 – 1551, copied from Imsser and extends it with his own tables for 1552 – 1556, calculated for the meridian of Venice. Not realizing that the author of the Tübingen tables was not Stöffler and that Stöffler's tables were not for Tübingen, he finds a number of confusing discrepancies, but in the end he notes that the time difference between Venice and Tübingen must be about 20 minutes. Since analysis of his year 1552 indicates that Pitati used a time difference of about 84 minutes from Toledo, this fits with my conclusion.

While the meridian issue obscures the conclusions for the sun, Pitati also notes that in all of Imsser's tables for the planets there is a tendency that ecliptic longitudes are slowly but surely lagging behind on the Alfonsine predictions, building up a considerable time difference around the middle of the year. He gives as an example that a Saturn-Mars conjunction, predicted for 25 May 1536, was not observed. And he concludes "Thus it must be said that almost all mathematicians agree that the Alfonsine Tables themselves come closest to the truth, especially for the motions of the other planets. (*It is permitted to some to make amendments about the fixed stars and the apogee*)".

It still seems strange that Imsser would deliberately have changed the apogee. What else could cause this apparent shift of some 10 - 12′? Did he re-use pre-calculated apogee tables for 1512 – 1531 that he might have inherited from Stöffler and used them for his own ephemerides of 1532 – 1551? Although that would explain it, yet it is hard to believe. It might also, in some other way, be an artefact of his own calculation routines. The question is intriguing and deserves further investigation, but right now we have no answer.

Martín Cortes' mysterious solar table

Martín Cortes' celebrated *Breve Compendio*[10] was printed in 1551 in Seville and an English translation was published in 1561 under the name *The Arte of Nauigation*. His introduction on the roundness of the Earth and the heavens is much shorter than in Nunez' *Tratado da sphera* but he goes into great detail on instrumentation.

For finding the *ECL* of the sun he gives a table for only one year and his procedure is the same as the nearly one century old *Kalendarium* method of Regiomontanus.[9] In a year that is not a leap year, you still look up the *ECL* under the proper calendar day and for dates from March 1 to the end of the year you subtract 1º from it. The table of the 'equation of the sunne', the shift needed for use in later years begins with 1545 and runs all the way to 1688.

The declination conversion table is the same as that of Nunez: the maximum declination is 23º 30′.

Cortes' root table turns out to be a clever construction. The basis is the year 1545, taken from Imsser's ephemerides. For January and February he gives Imsser's solar places with 1˚ subtracted. Then he inserts February 29 and continues with the numbers for March, unchanged. From 29 March till the end of the year, for no obvious reason, he adds 1′.

Consider the year 1545 itself to see how it works. A comparison with a real calculation with the Alfonsine Tables is shown in Figure 9(b). For 1545 the equation of the sun is given as 1˚. Adding this 1˚ restores Imsser's numbers for January and February, because Cortes had tabulated them with 1˚ subtracted. From March 1 onward, the instruction to first add 1˚ for the equation of the sun and then to subtract it again because it is not a leap year, restores his own table. The selective adding of an extra 1′ for 29 March – 31 December was not really necessary. It suggests a meridian 24 minutes in solar time more to the West, insufficient to bring it to Cadiz, which Cortes misleadingly tells us would be his meridian.

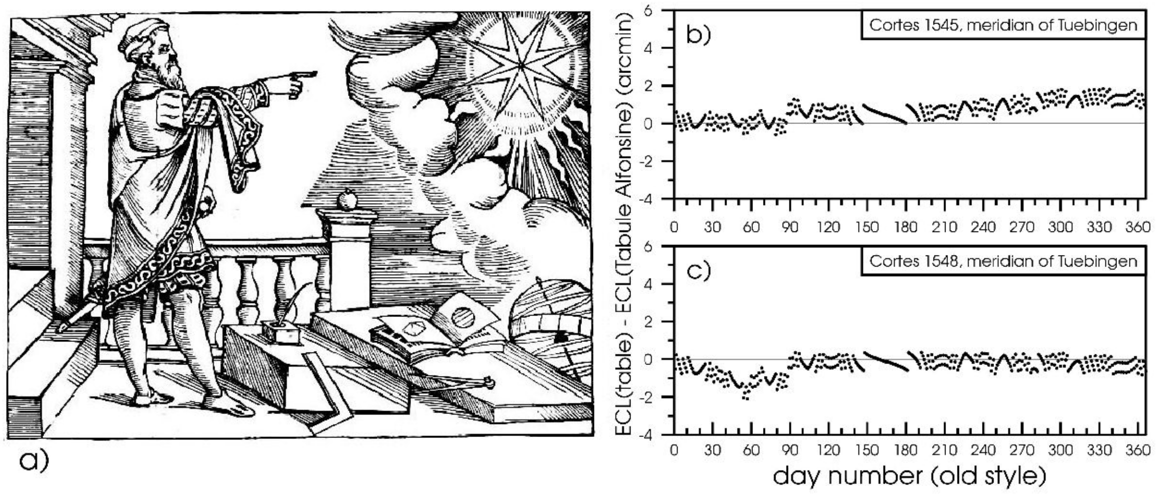

Figure 9. a) Illustration from Cortes' *Breve Compendio* that today, in a tile tableau, decorates the town hall of Bujaraloz, his native city. b) *ECL*'s for 1545, obtained by Cortes' recipe, analyzed with the Tabule Alfonsine for the meridian of Tübingen. c) Idem for 1548.

Figure 9(c) shows the solar longitudes for 1548, constructed again by Cortes' prescriptions. The reason that at least the part from April onward fits so nicely is a curious one. We have seen that the *Kalendarium* method leaves the apparent apogee, as it is embodied by the data, unchanged in date and time. In 1548, by the Alfonsine Tables and in the time of Tübingen, the sun passed its apogee on 13 June at 1:15:02. In 1545, the root year, that was at 13 June 7:07:53, just under six hours later. Since, however, Imsser had used an apogee, shifted by -12′, his table suggests a time of passage of 2:15:41, which in 1548 is only one hour off from the correct time.

Nicolaus Copernicus. The Prutenic Tables.

In 1543 the book of Nicolaus Copernicus, *De revolutionibus orbium coelestium* – on the revolutions of the celestial orbits – was printed.[3] The machinery behind the new, sun-centered model was different from that of the Alfonsine Tables. Erasmus Reinhold, an astronomer of great didactical skill, managed to formulate it in a new set of tables, the *Tabulae Prutenicae*, the Prutenic or Prussian tables, printed in 1551.[2] The name was fitting because Copernicus himself was from Prussia, born in Torun and later appointed canon in Frauenburg. And also because of the generous support of Albrecht, Duke of Prussia. Reinhold uses the Duke's birthday, 17 May 1490, whenever he illustrates the calculation procedures by an example.

Copernicus had tried to capture the evident time dependence of the obliquity in a formula that makes it range between 23º 52′ as a maximum and 23º 28′ as its minimum. By his estimate it would be close to its minimal value in the sixteenth century. A second important difference was the place of the apogee. Halfway the sixteenth century its ecliptic longitude would be about 91º.6 by the Alfonsine Tables. The new Prutenic Tables placed it more than 6º farther, at 97º.9. Third, the eccentricity of the Earth's orbit was taken about 20% smaller in the new tables than the value adopted by the Alfonsine Tables.

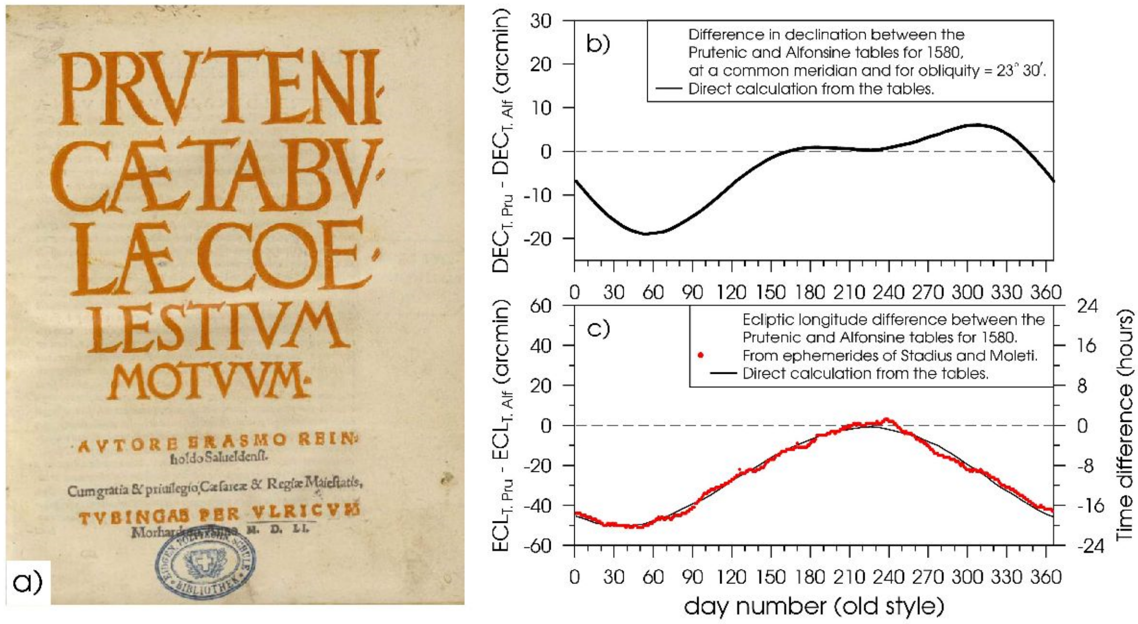

Figure 10. a) Front page of the 1551 edition of the Tabulae Prutenicae (copy of the ETH-Zürich). b) The difference in declination as derived from the Prutenic and the Alfonsine Tables. c) The corresponding difference in ecliptic longitude, illustrated by the difference between the *ECL*'s for 1580 of Stadius and Moleti. Irregularities in the data come from Stadius. The ephemerides of Moleti are much more consistent.

For the practitioner who wanted to calculate the positions of the sun and its declinations, there were significant changes compared with the Alfonsine Tables. Figure 10 illustrates this for the year 1580. Calculations with both tables are presented for a common meridian, which I took as that of Venice. Declinations, shown in Figure 10(b), are evaluated for a common obliquity, 23º 30′. The difference in *ECL* is further exemplified in Figure 10(c) by two existing ephemerides. The one for the Prutenic Tables is from Johannes Stadius (Jan van Ostaeyen), an extremely productive Flemish astronomer who made ephemerides for the years 1554 – 1606.[37] His home meridian was Antwerp. Iosephus (Giuseppe) Moleti had written ephemerides in the tradition of the Alfonsine Tables for the years 1564 – 1584 and for the meridian of Venice.[38] Figure 10(c) shows that the difference in *ECL* ranges between 0′ and 50′ over the span of one year, or a difference in predicted transit time of up to twenty hours. The vernal equinox comes about eighteen hours later in the Prutenic Tables than it does in the Alfonsine Tables. But since the Prutenic Tables place the apogee farther out, it gives a quicker pace along the ecliptic over spring and part of the summer, so that around mid-August its calculated *ECL* catches up with that from the Alfonsine Tables where after the situation reverses.

Figure 10(b) shows the difference in predicted declinations. Prutenic Tables values may become as much as 20′ smaller (i.e. more negative) than those for the Alfonsine Tables, This maximum difference falls around one or two weeks before the equinox.

The Regiments of the Sea

William Bourne was an inn-keeper from Gravesend and, as a former Royal Navy gunner, an expert in the art of shooting, on which he published several texts. He had a keen interest in mathematics, but no formal education. After thoroughly reading Cortes' *Arte of Nauigation* he concluded that it was too demanding for seafarers to expect them to find their declination by working their way through no less than three tables. After all, the Portuguese and the Spaniards had introduced four-year tables already half a century ago and declinations could be read directly from them without any calculation.

Bourne's first tabulations are in his *Almanacke and Prognostication for three yeeres*,[39] which were 1567 – 1569. A reprint for the years 1571 – 1573 repeats the same declination tables. Copies of this latter edition have survived. These declination tables are most likely based on the ephemerides of Johannes Stadius. In any case, Bourne's declinations fit slightly better with values derived from Stadius than with direct calculations via the Prutenic Tables. The maximum declination is taken as 23º 28′.

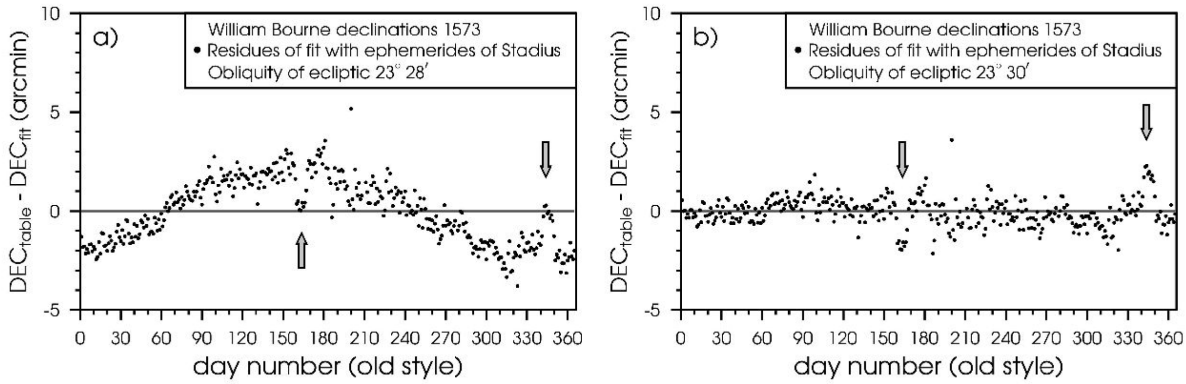

Figure 11. William Bourne's declinations for 1573 from his *Regiment of the Sea*, compared with their derivation from Stadius' ephemerides. a) Adopting 23º 28′ for the obliquity as Bourne claims he did. b) For an obliquity of 23º 30′. Arrows indicate the solstices where he has manipulated his declinations.

A few years later, Bourne writes his famous *Regiment of the Sea*,[40] which now contains declination tables for a four years. The first editions indicate that they were for 1573 -1576. In the later edition for 1577 – 1580 the years have been correspondingly changed in the table headers, but the declinations themselves are still the same.

Something funny is the matter with Bourne's declinations and this is illustrated in Figure 11 for the year 1573. Evidently not in the possession of a table for converting ecliptic longitudes to declinations for the 'new' obliquity of 23º 28′, he uses instead Martín Cortes' table, which is for 23º 30′. Figure 11(b) shows that this is what he has done. The fit is excellent but near the solstices, in the figure indicated by arrows, the declinations would become too large, larger than the newly adopted limit of 23º 28′, and Bourne has modified them to make them stay below this maximum. The left panel, Figure 11(a), shows that the fit is completely off when using a proper conversion table. I have done a similar analysis for his earliest table, that for 1567 and also for his last, which was calculated for 1576 (the one for 1580 is the same). The same feature shows up in all of them.

After Bourne's death the 'Regiment' was continued, from 1592 onward, by Thomas Hood, 'newly amended and corrected' and with new declination tables.[41]

Lucas Jansz. Waghenaer. New and old calendar.

Lucas Jansz Waghenaer was an experienced navigator from Enkhuizen. His seaman's pilot *Spieghel de Zeevaerdt*[42] was a folio size book, richly decorated and with many maps, charts and descriptions of coasts. Its first edition was printed in 1584 by Christoffel Plantijn in Leyden, later editions by Cornelis Claesz in Amsterdam. Of course it also had declination tables, and these were set in the new Gregorian calendar.

The English translation of Waghenaer's 'Spieghel', *The Mariners Mirrour*,[43] came out in 1588, with the declination tables re-organized after the old style Julian calendar. In the meantime, Cornelis Claesz had produced a handy pocket-book sized guide, without the charts but with the new style tables, under the title *Graetboecxken nae den nieuwen stijl*,[44] the 'booklet of degrees after the new style'. It's title page is shown in Figure 12(a), a scan of what may be the only surviving copy.

Not everyone was happy with the new tables. Many seafarers who possessed an astrolabe of their own, would have the calendar and the corresponding declinations engraved on the back of the instrument and now, by some new-fangled idea, this would suddenly be off by ten days. The protests were strong enough to make Claesz decide on the edition of another booklet, after the old style again.[45] In its foreword he writes with hardly hidden irritation that he does this to oblige his customers, but that for him personally this exercise has been far from profitable. And maybe as an expression of his opinion that time had left them behind, he gives them the oldest declination tables available, those from the Évora regiment. Evidently, these tables were still in use by the end of the sixteenth century. Also Willem Barents had them reprinted in his *Caertboeck vande Midtlandtse Zee* (Chart book of the Mediterranean).[46]

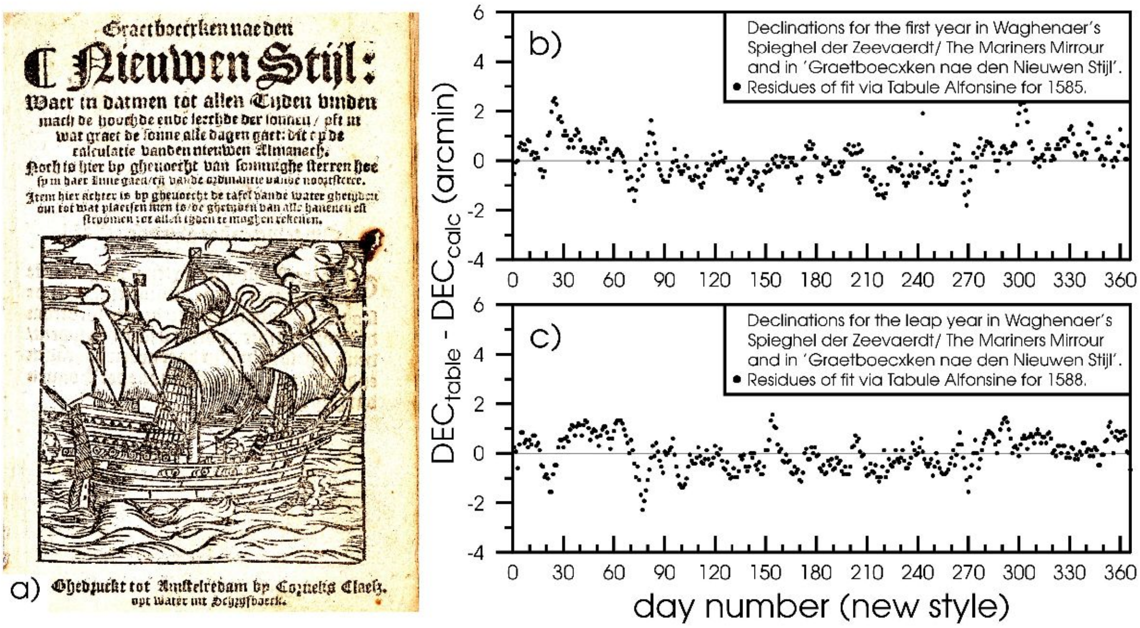

Figure 12. a) Title page of the *Graetboecxken nae den Nieuwen Stijl*. Scan from the University of Amsterdam copy. b-c) Declinations from Waghenaer's first and leap year. Residues upon a best fit via the Alfonsine tables for the indicated years 1585 and 1588.

In the introduction to his declination tables, Waghenaer tells us that they have been calculated for the years 1585 – 1588 by Adriaen Antoniszoon, land surveyor in the service of the Provinces of Holland. The maximum declination is the new value of 23º 28′. One might expect them to be based on the Prutenic Tables, but they are not. They have definitely been made from ephemerides in the Alfonsine tradition. Accepting the years that Waghenaer gives, 1585 – 1588, the fits are quite good and a best agreement is obtained for a meridian somewhat east from Enkhuizen. When assuming that the declinations would have been based on Nunez' (= Imsser's) *ECL* tables, shifted over 48 years, the agreement is again excellent. The meridian would be that of Tübingen and this is as good as one can get it. Fits for the first and the leap year are shown in Figures 12(b) and 12(c).

The Dutch translation of De Medina by Everaert/ Coignet

The most mysterious declination tables are those in the Dutch translation of De Medina's *Arte de Nauegar*.[31] Printed in Antwerp in 1580, it includes the translation by Marten Everaert and an extension on navigational techniques by Michiel Coignet. It is known that Willem Barents used it on his voyages to the North. A copy of this first edition has been found in their winter shelter on Novaya Zemlya. In 1589 the book was reprinted in Amsterdam, by Cornelis Claesz.

The maximum declination in the Dutch edition is 23º 30′, different from that in the original Spanish version and the French and English translations, where it is 23º 33′. In Coignet's chapter on navigation it is explained that this latter value might have been appropriate at the time of King Alfonso, but that the obliquity of the ecliptic had gradually decreased and was now 23º 28′. Learned men would speak of 23½ degree, just for convenience, and – not a very compelling argument - Marten Everaert had adjusted De Medina's tables accordingly.

But the new tables are not just a down-scaled version of the original ones. They are totally different and do not fit in the Alfonsine scheme, nor in that of the Prutenic Tables. I have long tried to find out what the plan behind them might have been, but with no success. I can only point out that the tables are inconsistent within themselves. To show this, consider that the length of the tropical year, just under 365 days and 6 hours, implies a shift of a quarter of a day between the declinations for two consecutive years when both are taken for the same day number. A natural consistency test is then:

$$\text{C-test statistic} = DEC2(\text{NDAY}) - [\tfrac{3}{4}DEC1(\text{NDAY}) + \tfrac{1}{4}DEC1(\text{NDAY}-1)], \qquad (2)$$

where *DEC*2 and *DEC*1 are the declinations of the second and first year, respectively and NDAY is the day number. Similarly, one may so compare the declinations of the third and the second year and of the leap year and the third year. If the tables have been made correctly, then within rounding errors this C-test statistic should vanish.

Figure 13 shows the results of the test for the new tables of Everaert and for the original ones of De Medina. While, as expected, the Medina tables pass the C-test with flying colors, Everaert's declinations do not. Also, there seems to be no regularity or systematics, because the three combinations of years that have been analyzed show patterns of the test results that are all different. Unfortunately, Waghenaer adopted these tables in his new book *Thresoor der Zeevaert* ('Treasure of Navigation')[47] to replace the much more consistent ones of his *Mirrour*.

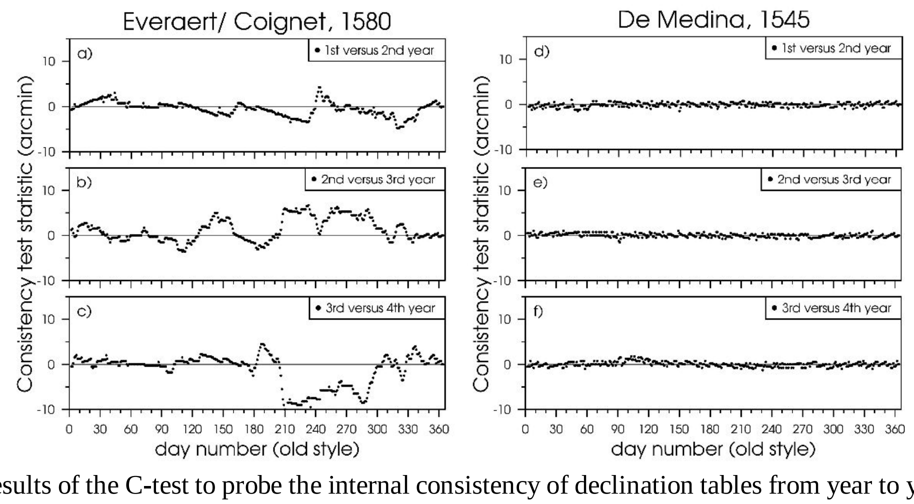

Figure 13. Results of the C-test to probe the internal consistency of declination tables from year to year. a,b,c) For the tables of Everaert in the Dutch translation of De Medina. d,e,f) For the original declinations in De Medina's *Arte de Navegar*.

Towards the 17th century. New astronomy, new tables

Excellent reviews of the English contributions to navigation have been written by Thomas Sonar[48] and by John Roche.[35] Above I have discussed the works of William Bourne and of Thomas Hood, who based their declination tables on the ephemerides of Johannes Stadius. Of special interest is the late sixteenth century work of Thomas Harriot. Although it was never printed and exists only in manuscript form,[49] it is witness of a growing discontent with the Prutenic Tables. Harriot decided to do astronomical observations of his own and use those to produce declination tables for the years 1593 – 1596. Sir Walter Raleigh used them on his voyage to Guyana in 1595.

Roche has made an in-depth analysis of these tables, showing that Harriot adopted a place of the apogee halfway the 6th degree of Cancer, intermediate between the values of the Prutenic and the Alfonsine tables. Also the eccentricity is different and intermediate between what these tables prescribe and Harriot chose the obliquity as 23º 31′. I have made a least-square regression on the full table for the year 1593, with as adjustable parameters the time of the equinox, the eccentricity and the place of the apogee. The result is: equinox at $00^h 34^m \pm 2^m$ PM on 10 March, apogee at 6º 12′ ± 4′ in Cancer and eccentricity = 0.0360 ± 0. 0006, in complete agreement with Roche, but with smaller statistical errors. The standard deviation per data point is only 0′.32, illustrating how carefully the tables were made.

It seems fitting that we end this survey with the new revolution in astronomy that was initiated by Tycho Brahe and finalized by Johannes Kepler. Tycho's collected notes were published by his heirs in 1602 under redaction of Kepler as *Astronomiae Instauratae Progymnasmata*.[50] It contains full solar tables for 1572 and 1573 and auxiliary tables by which solar longitudes can be computed for later years. The obliquity is 23º 31′.5.

In the winter of 1595 – 1596 the young Willem Jansz Blaeu worked with Tycho on the island Hven as one of his many assistants. His interest was in making maps and globes, a profession that would make him world famous. Blaeu had of course access to the new star catalogue. It is believed that, during his stay, he copied it and took it back to Holland, where he used it to make his celestial globes.[51,52]

Later, in 1608, Blaeu published his navigational handbook *Het Licht der Zeevaert*[53] and in 1612 the English translation *The Light of Navigation*.[54] In his introduction he writes that the declination tables are fully new and have been based on the ephemerides of Tycho Brahe. With the Progymnasmata now printed and published, it was of course perfectly legal for him to do so, but there is evidence that he may have copied them already during his stay on Hven.

Figure 14 shows the residues of Progymnasmata-based fits of Bleau's declinations for the leap year and for the first year under two scenario's. Figures 14(a) and 14(b) assuming that they are for 1608 and 1609, the years for which Bleau's tables were first published. This is obviously not the case. The sine-like shape around the summer solstice is typical for the equinoxes in the calculation coming earlier than in the data.

Figures 14(c) and 14(d) show that a shift of only twelve years from the Progymnasmata tables is required. That would date them as for 1584 and 1585, which of course they are not. The most likely scenario is that, while on Hven , indeed twelve years before the printing of his *Licht der Zeevaert*, Bleau also copied Brahe's solar tables and must have thought them recent.

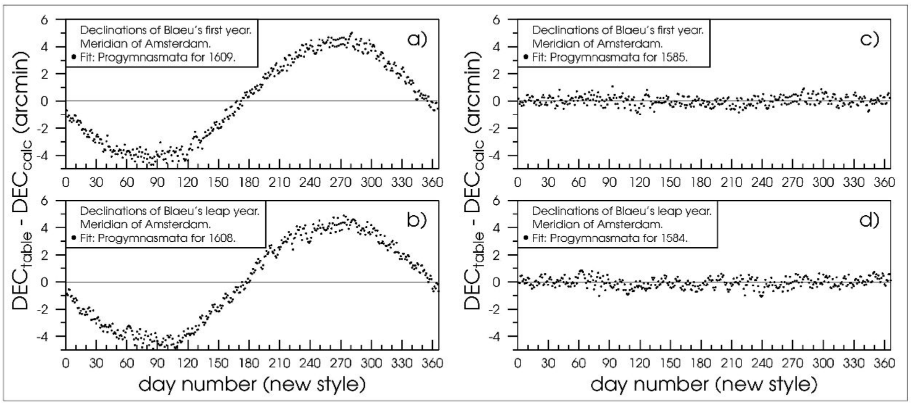

Figure 14. The first and the leap year of Willem Jansz Blaeu's *Licht der Zeevaert/ Light of Navigation,* compared with values derived from Brahe's *Progymnasmata.* (a) and (b) assuming a shift of 36 years from root years 1572 and 1573. (c) and (d) assuming a shift of 12 years.

Absolute accuracy of declination tables from comparison with modern calculations

It is an important question – especially for nautical history writing – to establish in retrospect the absolute accuracy of the declination tables that were most widely in use. When a sixteenth century navigator would find his latitude by taking the height of the sun at local noon, by how much could a tabulated declination be off from reality? Modern ephemerides programs like the one of Jet Propulsion Laboratory[55] and all major planetarium programs, SkyMap[56], Cybersky[57], StarCalc[58] and others, use the same professional software and agree in full on all observables.

Figure 15 shows the differences of the tabulated declination tables with modern calculations, for the year and the meridian for which they were made. All of them are for a first year after a leap year.

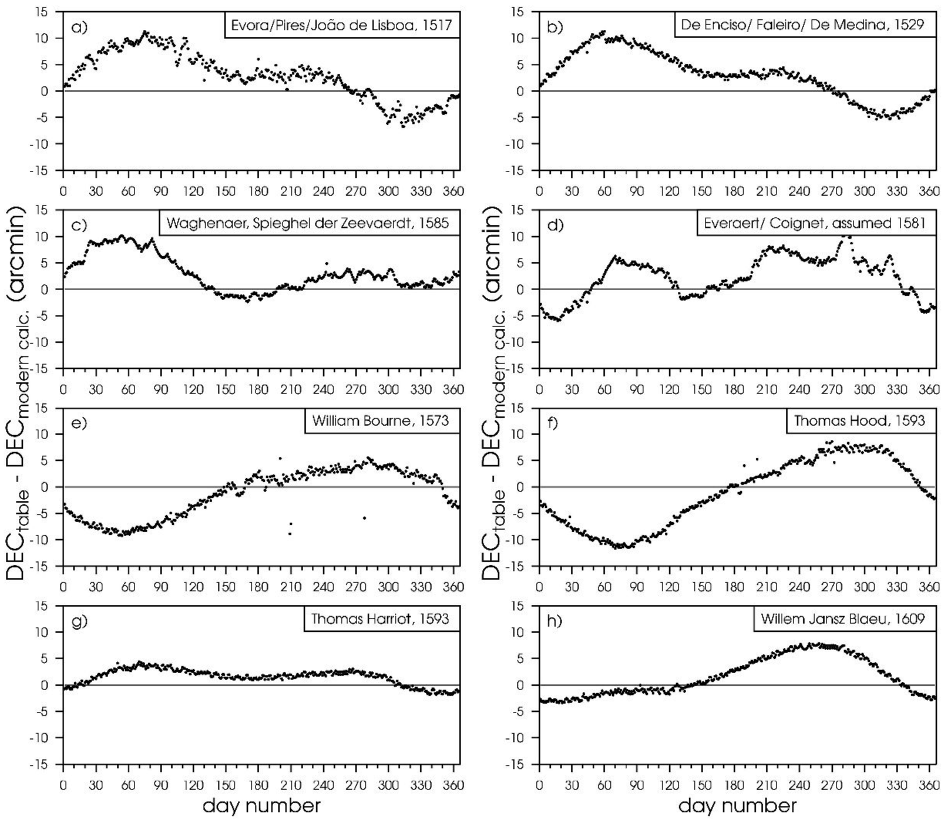

Figure 15. Differences of popular tabulated declination tables and the corresponding modern calculation.

Figures 15(a)-(c) give tables in the Alfonsine tradition, The ones of Évora/Pires/Joao de Lisboa, those of De Enciso/Faleiro/De Medina and the ones of Waghenaer's *Spieghel der Zeevaerdt*, which are also in the *Graetboecxken nae den Nieuwen Stijl*. Figures 16(e)–(f) present the results for the tables of William Bourne and of Thomas Hood which are Prutenic.

The most prominent deviation is just before the vernal equinox and for Alfonsine Table-based declinations (a-c) it amounts to around 10′ to the positive side. For declinations made after the Prutenic Tables (e-f) the maximum difference is also some 10′, but to the negative.

The tables of Everaert and Coignet in the Dutch translation of De Medina (Figure 16-d) fit in no scheme, as shown above, but their general agreement or disagreement with a modern calculation is not better or worse than for typical Alfonsine or Prutenic tables.

Closest to a modern calculation comes Thomas Harriot (Figure 16-g), who based his declination tables upon measurements of his own. In particular, his placing of the equinox and the apogee is very close. Roche[35] has made a similar comparison for the full four-year cycle 1593-96 sampling in steps of ten days. His conclusion is that nowhere Harriot was off from a modern calculation by more than 4′.

Figure 16(h), finally, shows the differences for the table of Willem Jansz. Blaeu, presented as made for 1609. As discussed above, these tables are older than Blaeu seems to have been aware of. The prominent deviation in the second half of the year, rising to about 7′, is mostly due to his erroneous extrapolation.

This brings us to a second source of error: the use of tables in later and sometimes much later years than for which they were written. Maybe the longest use of all was for the Portuguese tables for 1517-1520 that we find in the Évora regiment, the tables of André Pires and of João de Lisboa. They were still in use in the late sixteenth century. Figure 16 shows what happens when using these tables in 1517, for which they were written, in 1557, forty years later, and in 1597, the year of the wintering on Novaya Zemlya and the return voyage in two open boats.

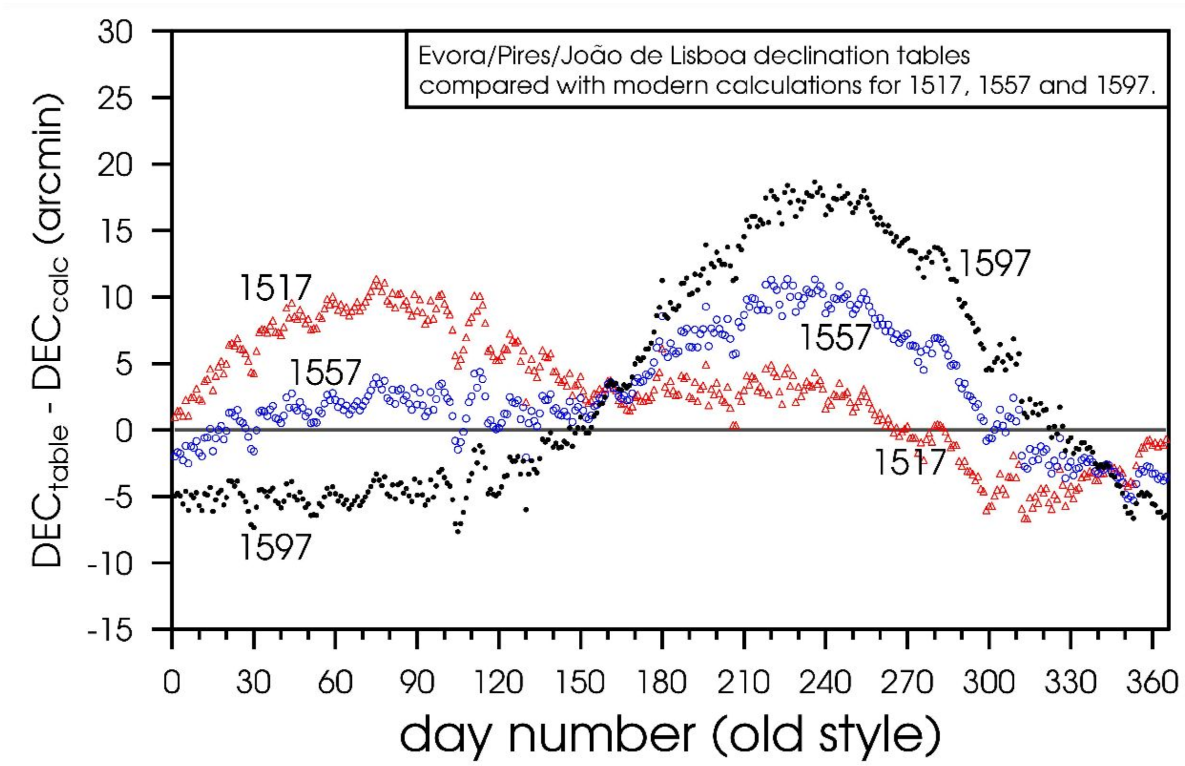

Figure 16. Differences of the Évora/Pires/João de Lisboa tables with modern calculations for 1517, 1557 and 1597.

For 1517 the deviation is typical for the Alfonsine scheme. The largest difference is found in the first half year and it can become around 10′. Forty years later, in 1557, the equinox falls about seven hours earlier, closer to where it was placed by the Alfonsine Tables, and the deviation in the first half year has almost disappeared. At the same time it has grown to around 10′ in the second half year. Thus, forty years later, the overall accuracy of the tables is still the same as it was in the beginning and in between the deviations are actually smaller. After these forty years, the maximum in the second half year keeps growing and in 1597 the error may become as large as 17′.

Summary and conclusions

Astronomical and declination tables of the fifteenth and sixteenth century have been analyzed via direct comparison with the algorithms of the Alfonsine Tables and the Prutenic Tables and in some cases by least square regression wherein the time of the vernal equinox, the eccentricity of the solar orbit and the place of the apogee are treated as adjustable parameters. In quite a number of cases the conclusions differ from existing history writing and I list them here.

1) The often used quick method of finding solar longitudes by shifting the data of a single root year is adequate for getting the equinox right, but leaves the apparent apogee where it was in the root year, both in calendar date and time.

2) Zacut’s tables for 1474 and for the first half 1475 are just shifted versions of 1473, causing an incorrect apparent placing of the apogee.

3) The solar tables for 1537 – 1540 in Pedro Nunez’ *Tratado da sphera*, have been copied from those of Philipp Imsser, which usually, but incorrectly, are attributed to his predecessor Johannes Stöffler. Imsser used a place for the apogee, slightly different from the Alfonsine prediction. This was later pointed out by Pietro Pitati.

4) The solar table in Martín Cortes' *Breve Compendio* is a construction, based on the 1545 table of Imsser. Cortes' statement that his table is for the meridian of Cadiz, is incorrect.
5) William Bourne has based the declination tables in his *Regiment for the Sea* on the ephemerides of Johannes Stadius, but lacking a proper conversion table for the 'new' obliquity of 23º 28′, he used Cortes' table for 23º 30′ instead and manipulated the results near the solstices.
6) The declination tables of Waghenaer's *Spieghel der Zeevaerdt/ Mirrour of Navigation* have still been made after the Alfonsine Tables, however using an obliquity of 23º 28′ as prescribed by the Prutenic tables.
7) In the Dutch translation of De Medina's *Arte de nauegar* the new declination tables that replace the original ones fit in no scheme and they are inconsistent within themselves.
8) There is evidence that Willem Jansz. Blaeu may have copied Tycho Brahe's solar tables on which he based his declination tables in *Het Licht der Zeevaert/ The Light of Navigation*, already during his stay on Hven in 1595 – 1596.

In addition the absolute accuracy of the declination tables has been assessed by comparing them with calculations based on modern astronomical software. As long as tables were used in the years for which they were compiled the deviation is generally within 10′. Using them in later years does not immediately make the Alfonsine based tables worse, at first it makes them even better. It takes about forty years before the overall deviations become again as large as they were in the beginning. Thereafter, the disagreement keeps growing in the second half of the year.

For Prutenic Table based declinations the situation is worse. They place the equinox too late and for later years that will be later yet. The deviation in the first half year gets more negative than it already is. In the second half year, where it is positive already, it increases further. Thus, while Alfonsine declination tables may be used for about forty years without loss of accuracy, Prutenic declination tables are most accurate in the years for which they were written and lose accuracy with every later year. But then, none of such Prutenic tables have been in use as long as the Évora tables.

Note on Contributor
Siebren van der Werf is a retired nuclear physicist of the University of Groningen, The Netherlands. His current interest and work is on the history of navigation and on refraction in the atmosphere and computer simulations of its anomalies, such as the fata morgana and the Novaya Zemlya phenomenon.

Notes

1. G.J. Toomer, *Ptolemy's Almagest* (Princeton, NJ: Princeton University Press, 1998).
2. E. Reinhold, *Prutenicae Tabulae Coelestium Motuum* (U. Morhard,Tubingen, 1551).
3. N. Copernicus, *De revolutionibus orbium coelestium libri VI* (J. Petreius, Nuremberg, 1543).
4. J. Danck de Saxonia, *Tabulae astronomicae Alphonsinae* (E. Ratdolt, Venice, 1483).
5. J.L Santritter, *Tabule astronomice Alfonsi Regis* (J. Hamman, Venice, 1492).
6. E. Rosen, J. Regiomontanus, *Complete Dictionary of Scientific Biography* (Charles Scribner's Sons, Detroit, 2008).
7. J. Regiomontanus, *Ephemerides 1474–1506* (Nuremberg, 1474). Reprinted: *Ephemerides 1492–1506* (ed. Erhard Ratdolt, Augsburg, 1492).
8. J. Regiomontanus, *Tabulae directionum profectionumque* (J. Engel, Augsburg, 1490).
9. J. Regiomontanus, *Kalendarium* (Nuremberg, 1474). Many reprints by E. Ratdolt, Venice.

10. M. Cortes, *Breve Compendio de la sphera y de arte de nauegar …*, (A. Alvarez, Seville, 1551). *The Arte of Nauigation …*, translated into English by R. Eden (London, 1561).
11. A. Zacut, *Almanach Perpetuus coelestius motuus,* translated from Hebrew into Latin by J. Vizinho (Leiria, 1496).
12. A. Zacut, *Almanach perpetuus exactissime nuper emendatum* (P. Liechtenstein, Venice, 1502).
13. J. Chabas and B.R. Goldstein, "Astronomy in the Iberian Peninsula: Abraham Zacut and the
Transition from Manuscript to Print," *Transactions of the American Philosophical Society*, 90(2), 2000, p. 104.
14. J. Bensaude, *L' Astronomie nautique au Portugal a l'epoque des grandes decouvertes* (Bern,
1912).
15. J. Bensaude, *Regimento do estrolabio e do quadrante: tractado da spera do mundo*, reproduction
fac-simile du seul exemplaire connu appartenant a la Bibliotheque Royale de Munich
(C. Kuhn, Munich, 1914).
16. L.M. De Albuquerque, *Os guias nauticos de Munique e Evora* (Junta de Investigacoes do Ultramar, Lisbon, 1965).
17. I. De Sacrobosco, *Sphaere Mundi*; G. Purbachius, *Theoricae novae planetarum*; J. Regiomontanus, *Disputationes contra Cremonensia in planetarum theoricas deliramenta praefatio* (E. Ratdolt, Venice, 1485).
18. L.P. Da Silva, *Obras Completas de Luciano Pereira da Silva Volume 2* (Faculty of Sciences of the University of Coimbra, Coimbra, 1945).
19. W. Koberer, "On the Attempts to Assess the Accuracy of the Astrolabe," *The Mariner's Mirror*, 100(2), 2014, pp. 198–203.
20. R. Knox-Johnston, "Practical Assessment of the Accuracy of the Astrolabe," *The Mariner's*
Mirror, 99(1), 2013, pp. 67–71.
21. N. De Hilster, "Observational Methods and Procedures for the Mariner's Astrolabe," *The Mariner's Mirror*, 100(3), 2014, pp. 261–81.
22. J. De Lisboa, *Livro de Marinharia* (Lisbon, c. 1516). The digitized manuscript is accessible
via <digitarq.dgarq.gov.pt>
23. V. Fernandes, *Reportorio dos tempos* (Lisbon, 1518).
24. A. Pires, *Livro de Marinharia*, Bibliotheque Nat. de Paris, Codice 44340 (Ms. Port., 40), pp. 1–37.
25. L.M. De Albuquerque, *O livro de marinharia de Andre Pires* Junta de Investigacoes do Ultramar, Lisbon, 1963).
26. M.F. De Enciso, *Suma de Geographia* (J. Cromberger, Seville, 1530).
27. F. Faleiro, *Tratado del esphera y del arte del marear* (J. Cromberger, Seville, 1535).
28. P. De Medina, *Arte de Nauegar* (F. Fernandez, Valladolid, 1545).
29. P. De Medina, *L'art de Naviguer*, translated into French and extended by N. De Nicolai (G. Roville, Lyon, 1554).
30. P. De Medina, *The Arte of Navigation*, translated into English by J. Frampton (T. Dawson, London, 1581).
31. P. De Medina, *De Zeevaert oft Conste van ter Zee te varen*, translated into Dutch by M. Everaert and extended by M. Coignet (H. Hendricksen, Antwerp, 1580).
32. P. Nunez, *Tratrado da sphere* (M. da Costa, Lisbon, 1537).
33. Philipp Imsser, *Ephemeridum opus Ioannis Stoefleri Justingesis mathematici a capite anni*

redemptoris Christi MDXXXII in alios XX proxime subsequentes, ad veterum imitationem accuratissimo calculo elaboratum (H. Morhard, Tubingen, 1531).
34. J. Stoffler and J. Pflaum, *Almanach nova plurimis annis* (J. Reger, Ulm, 1499), Reprinted (P. Liechtenstein, Venice,1507).
35. J.J. Roche, "Harriot's Regiment of the Sun …," *The British Journal for the History of Science*,14(48), 1981, pp. 245–61.
36. P. Pitatus, *Almanach novum Petri Pitati Veronensis* (L. A. Giunta, Venice, 1541).
37. J. Stadius, *Ephemerides novae.* Many printings: 1554–1570 (A. Birckmann, Cologne, 1556); 1554–1576, (A. Birckmann,Cologne, 1560); 1562–1567 (A. Birckmann,Cologne, 1570); 1554–1581 (A. Birckmann,Cologne, 1581); 1583–1606;(P. Tinghi, Lyon, 1585).
38. I. Moleti, *Ephemerides 1564–1584* (F. Senese, Venice, 1564).
39. W. Bourne, *An Almanacke and Prognistication for three yeeres* (T. Purfoote, London, 1571).
40. W. Bourne, *A Regiment for the Sea* (T. Hacket, London, 1573) and (T. Dawson, London, 1573); Reprinted (T. East for J. Wight, London,1577.
41. W. Bourne, *A Regiment for the Sea*, corrected and extended by T. Hood (T. East for T. Wight, London, 1593).
42. L.J. Waghenaer, *Spieghel der Zeevaerdt* (C. Plantijn, Leyden, 1584).
43. L.J. Waghenaer, *The Mariners Mirrour* (A. Ashley (transl.), London, 1588).
44. C. Claesz, *Graetboecxken nae den Nieuwen Stijl* (Amsterdam, 1587). The library of the University of Amsterdam has an incomplete copy of this rare work, [OTM: OK 61–141 (3)].
45. C. Claesz, *Graetboeck nae den Ouden Stijl* (Amsterdam, 1595). A very well preserved copy
is in the collection of the Maritime Museum of Rotterdam, inventory no. 29481.
46. W. Barents, *Nieuwe Beschryvinghe ende Caertboeck vande Midtlandtsche Zee* (C. Claesz, Amsterdam,
1595).
47. L.J. Waghenaer, *Thresoor der Zeevaert* (F. van Raphelengien, Leyden, 1592).
48. T. Sonar, "The 'Regiments' of Sun and Pole Star: On Declination Tables in Early Modern England," *International Journal on Geomathematics*, 2010, 1, pp. 5–21.
49. T. Harriot, BM Add.MS 6788. ff. 409–420 (original: 205r – 210v).
50. T. Brahe, *Astronomiae Instauratae Progymnasmata* (Heirs of T. Brahe, Prague, 1602).
51. V.E. Thoren, *The Lord of Uraniborg* (Cambridge University Press, Cambridge, 1990).
52. R. Van Gent, "De nieuwe sterren van 1572, 1600 en 1604 op de hemelglobes van Willem Jansz Blaeu," *Caert-Thresoor*, 12, 1993, pp. 40–6.
53. W.J. Blaeu, *Het Licht der Zeevaert* (Amsterdam, 1608).
54. W.J. Blaeu, *The Light of Navigation* (Amsterdam, 1612).
55. Jet Propulsion Laboratory, <http://ssd.jpl.nasa.gov/horizons.cgi#top>.
56. C. Marriott, *SkyMap Astronomy Software* <http://skymap.com/>.
57. S.M. Schimpf, *Cybersky, Astronomy Software for Windows* <http://www.cybersky.com/>.
58. A. Zavalishin, *Starcalc* <http://homes.relex.ru/~zalex/main.htm#about